

Probabilities of Compound Events

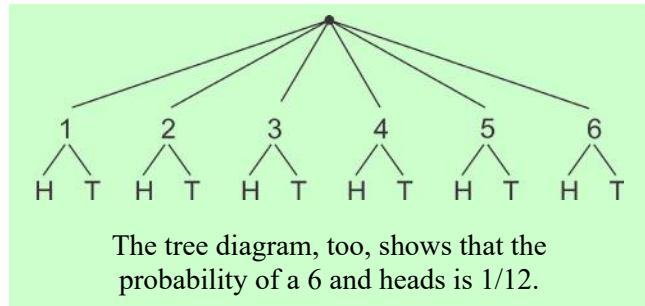
(This lesson is optional.)

Up to now we've been looking only at **simple events**, events that require just a single calculation of probability. A **compound event** is an event that consists of two or more simple events. If the outcome of one event does not affect the outcome of another, the events are said to be **independent**. If the compound event consists only of independent simple events, then it is very easy to calculate the probability of the compound event: we simply multiply the probabilities of the individual simple events. The examples will make this clear.

Example 1. You roll a die and toss a coin. What is the probability of rolling a 6 and getting heads?

$P(6)$ is $1/6$ and $P(\text{heads})$ is $1/2$. Clearly, whether you get heads or tails on the coin does not affect what you get on the roll, so the two events are independent. Therefore, we can multiply the two probabilities:

$$P(6 \text{ and heads}) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

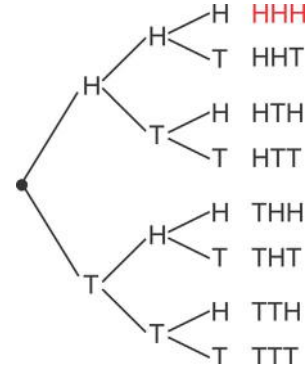


Example 2. You toss a coin three times. What is the probability of getting heads every time?

These three events—toss a coin, toss a coin, toss a coin—are independent. Getting heads on one toss doesn't affect whether you get heads or tails on the next.

$$P(\text{heads}) = 1/2. \text{ Therefore, } P(\text{heads and heads and heads}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.$$

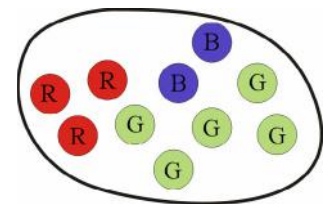
You can also see this result in the tree diagram. Only one outcome out of the 8 is "HHH."



Example 3. A bag has three red marbles, two blue marbles, and five green marbles. You take out a marble and put it back. Then you take out a marble again and put it back. What is the probability of getting first a red marble and then a blue one?

Again, we simply multiply the individual probabilities:

$$P(\text{red, blue}) = \frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100} = \frac{3}{50}.$$



1. You toss a coin three times.
 - a. What is the probability of getting tails, then heads, then tails?
 - b. What is the probability that you get heads on your second toss?
 - c. Use the tree diagram in Example 2. What is the probability of getting heads twice and tails once in three tosses? Note that they can be in any order, such as THH or HTH.