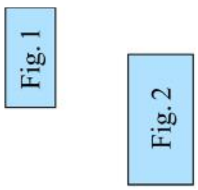
Similar Figures, Part 1

Definition: We call two figures **similar** if there is a sequence of transformations (translation, reflection, rotation, dilation) that maps one figure to the other.

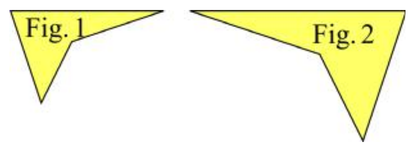
Figures that are dilations of each other are similar, no matter where they are located in the plane, or whether they have been rotated or reflected.

Example 1. A sequence of a dilation, a rotation, and a translation maps the smaller tree to the bigger tree. The two figures are similar.

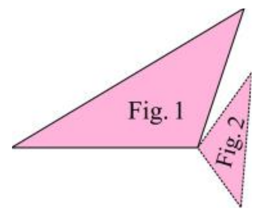
1. State the transformations that can map figure 1 to figure 2. You don't need to include details about the transformations, such as the scale factor, the exact line of reflection, or the amount of translation or rotation.



a.



b.

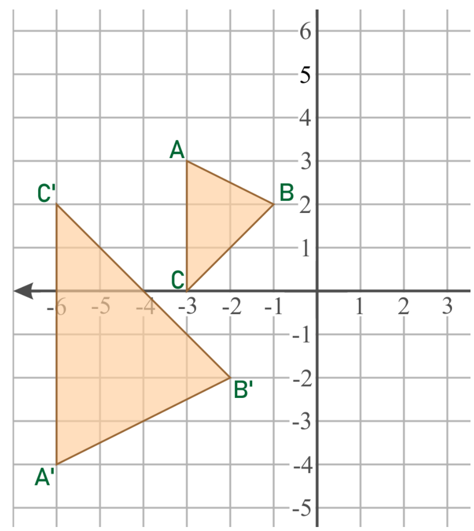


c.

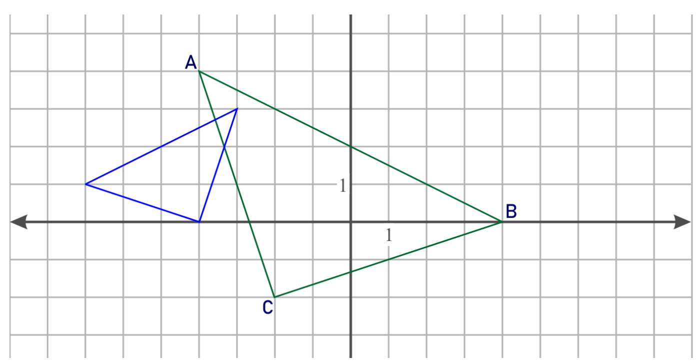
2. Henry says that triangle ABC is similar to triangle A'B'C' because △ABC can be mapped to △A'B'C' by first dilating △ABC with origin as center and with the scale factor 2, and then reflecting the resulting figure in the horizontal line at $y = 1$.

   Harry says that's not true, that instead, △ABC is first reflected in the x-axis, and then dilated with B' as center point, with scale factor 2.

   Whose proof is correct, or are both correct?



3. Show that the two triangles are similar by describing a sequence of transformations that could map △ABC to the smaller triangle.



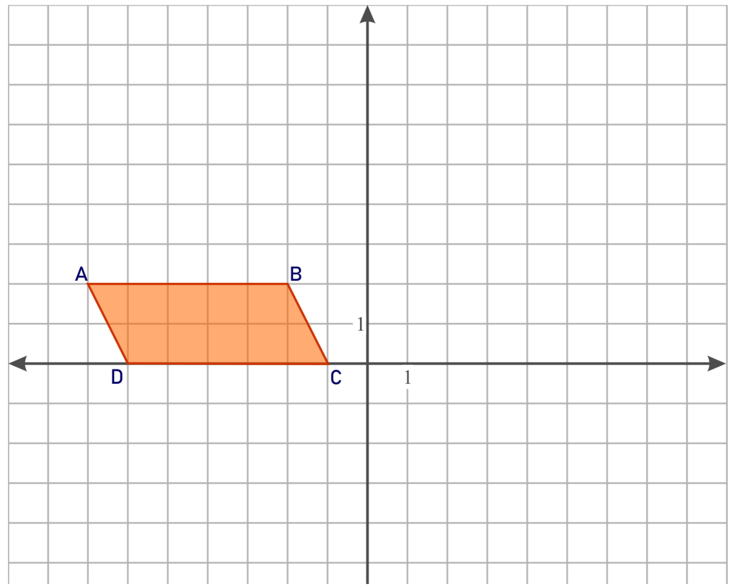
4. Parallelogram ABCD underwent the following transformations :

1. A 90° rotation clockwise around the origin.

2. Translation 3 units to the right and 4 units down.

3. Dilation centered at B" with scale factor 1/2.

What are the coordinates of the image of point D (D"') after all these transformations?



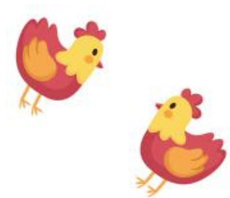
5. Triangle PQR underwent two transformations. Study the coordinates to find out the details about each transformation. Then describe each transformation in detail. Use grid paper if necessary.

Original figure	Transformation 1	Transformation 2
P(−4, 0)	P'(−1, 0)	P"(1, −4)
Q(0, 4)	Q'(0, 1)	Q"(2, −3)
R(−8, 4)	R'(−2, 1)	R"(0, −3)

Transformation 1:

Transformation 2:

6. **a.** If two figures are congruent, are they also similar? Explain your reasoning using the definition of similarity.

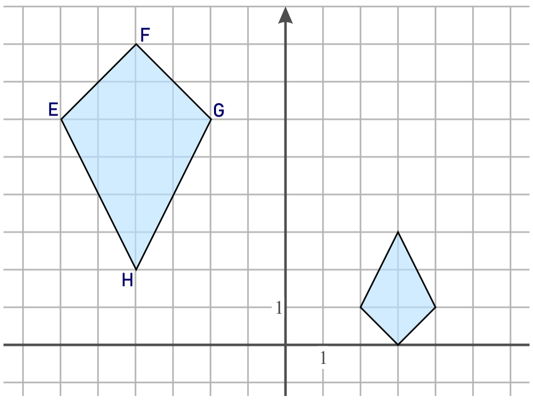


b. Is it true that two similar figures must also be congruent? Why or why not?

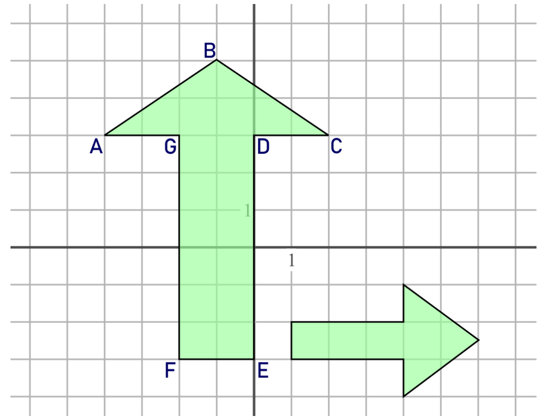
7. Figure EFGH underwent a dilation, then a reflection. Study the coordinates to find out the details about each transformation, then fill in the missing coordinates. Use grid paper if necessary.

Original figure	Dilation	Reflection
E(−1, −1)	E'(−5, −2)	E"(−5, 2)
F(3, 0)	F'(3, 0)	F"(3, 0)
G(3, −2)	G'(3, −4)	G"(____ , ____)
H(2, −2)	H'(____ , ____)	H"(____ , ____)

8. Are the two figures similar? If yes, give a proof of that by giving a sequence of transformations (with details) that maps one to the other.

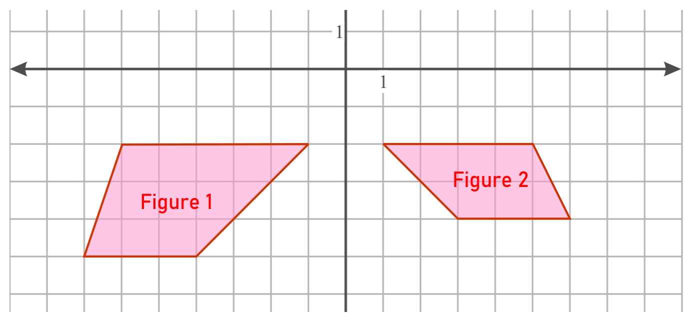


a.



b.

9. Which statement is true?



(1) A reflection in the *y*-axis, followed by a dilation, will transform Figure 1 to Figure 2, proving the two are similar.

(2) There is no sequence of transformations that will map Figure 1 to Figure 2, making the two figures neither congruent nor similar.

(3) A rotation 180 degrees about the origin, followed by a translation, followed by a dilation, will transform Figure 1 to Figure 2, proving the two are similar.

(4) A translation, then a reflection will make Figure 1 map to Figure 2, proving they are congruent.

Puzzle Corner

Figure WXYZ underwent two mystery transformations. Study the coordinates to find out the details about each transformation, then fill in the missing coordinates. Grid paper can help.

Original figure	Transformation 1	Transformation 2
W(−6, 0)	W'(2, 0)	W"(2, 0)
X(−3, 1)	X'(−1, 1)	X"(3, 3)
Y(−3, −3)	Y'(−1, −3)	Y"(−1 , 3)
Z(−6, −2)	Z'(____ , ____)	Z"(____ , ____)